Green Acres is the Place to Be!

I. UNIT OVERVIEW & PURPOSE:

Geometry from a farm perspective! Students will help a farmer make some important decisions based on mathematics – determining how to harvest a crop, maximizing area and volume (storage), designing structures, deciding if figures are similar and realizing the similarity ratios that exist in similar figures. The economics of certain situations are also addressed.

II. UNIT AUTHORS:

Amanda Lucas, Craig County High School, Craig County Public Schools
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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry – Three Dimensional Figures

V. OBJECTIVES:

SOL: G.13, G.14

- Students will differentiate between the measurements of perimeter/circumference, area, lateral area, surface area, and volume.
- Students will relate the above measurements to the appropriate two- or three-dimensional figure(s) in a real-world setting and apply the use of appropriate units.
- Students will calculate the perimeter/circumference, area, lateral area, surface area, and volume of two- and three-dimensional figures using formulas in real-world applications.
- Students will compare measurements between different geometric figures.
- Students will identify similar three-dimensional geometric figures and show the scale factor (proportion) that exists between dimensions of these figures.
- Students will solve proportions.
- Students will determine and use the similarity ratio (scale) of a dimension of two similar three-dimensional geometric figures to also determine the similarity ratio of the surface area and volume of the figures and vice versa.
- Students will calculate the surface and volume of the similar figure using only the similarity ratio.

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in collaboration with
Mr. Michael Bolling, Virginia Department of Education

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE.6 – The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

MPE.7 – The student will use similar geometric objects in two- or three-dimensions to

a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

MPE.34 – The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as proofs.

VII. CONTENT:

Mathematically this unit specifically addresses finding the perimeter/circumference and area of two –dimensional geometric figures as well as lateral area, surface area, and volume of three-dimensional geometric figures. The topic of similar figures will also be used to introduce and then explore the relationship between length, surface area, and volume of similar three-dimensional geometric figures.

The overall goals of this unit are to make students aware of the mathematical concepts used by a farmer (someone who students do not think would use math) and to lead students to the conclusion that special relationships (ratios) exist between the dimensions and calculations of three-dimensional figures.

VIII. REFERENCE/RESOURCE MATERIALS:

- Focus Activities, Student Exploration Worksheets and Exit Slip Assessments are needed for all five lessons. Please see individual lessons.
- Graphing calculators will be required.
- In addition students will need access to a set of two-dimensional and three-dimensional geometric models on a daily basis to help visualize the shapes presented in the unit.
- Internet accessible computer may also be used.
- 3-D Shape internet videos
    - http://www.youtube.com/watch?v=K9L9l86N-xM&list=PL8C80DD82B20625BE
    - http://www.watchknowlearn.org/Video.aspx?VideoID=36007&CategoryID=1010
    - http://www.teachertube.com/viewVideo.php?video_id=24955

- Geometry SOL Formula Sheet
  www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf
- Nets of 3-D Geometric Figures
  http://edgalaxy.com/journal/2012/8/5/great-collection-of-3d-shape-nets-to-download-and-print.html

IX. PRIMARY ASSESSMENT STRATEGIES:

Classroom Explorations
Exit Slips (including reflection)
Homework Assignments (optional)

X. EVALUATION CRITERIA:

Students will participate in daily class explorations and discussions and complete the exit slip and homework assignments. A rubric is attached to assist in the assessment process. The same rubric will be used on each day of the lesson.

XI. INSTRUCTIONAL TIME:

This unit is intended to take five days in a 90 block.

Lesson 1 – Square vs. Round

Strand
Geometry -- Three Dimensional (3-D) Figures

Mathematical Objective(s)
- Students will differentiate between the measurements of perimeter/circumference, area, lateral area, surface area, and volume.
- Students will relate the above measurements to the appropriate two- or three-dimensional figure(s) in a real-world setting and apply the use of appropriate units.
- Students will calculate the perimeter/circumference, area, lateral area, surface area, and volume of two- and three-dimensional figures using formulas in real-world applications.
- Students will compare measurements between different geometric figures.

Mathematics Performance Expectation(s)
- MPE.6 – The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

Related SOL
- G.13

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Draw and construct representations of two- and three- dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

Materials/Resources
- Classroom Set of Graphing Calculators
- Access to Smart Board or laptop and projector (with internet connection).
- Copies of Focus Activity (Geometric Figure ID Worksheet – What Shape Am I?)
- Copies of Student Exploration Packet (Square vs. Round)
- Copies of Assessment (Exit Slip – Lesson 1 – Square vs. Round)
- Copies of HW Assignment (optional).
- Set of 3-D models. (Cylinder, Triangular Prism, Rectangular Prism, Sphere, Hemisphere, Pyramid, Cone, Cube)
- Copies of Geometry SOL Formula Sheet.
  www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf

Assumption of Prior Knowledge

- Students should be able to draw on prior knowledge of both two-dimensional and three-dimensional geometric figures.
- Students should be able to identify geometric figures by appearance even when combined. Students should be operating on the "analysis" level of the Van Hiele scale with respect to two- and three-dimensional geometric figures.
    - Students can recognize and name properties of the figures.
    - Properties cannot be tied to other properties that exist.
    - Students can reason informally about the figures.
- Students will begin to compare measurements of two non-similar three-dimensional geometric figures. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly name prisms and will not be able to differentiate between lateral area, surface area, and volume.
- The priority for this lesson is for students to understand how to calculate perimeter/circumference, area, lateral area, and surface area and explain the differences in the measurements formed and learn how they are applied in real world situations.
- Relevant contexts: Analysis and impact of sizes and shapes of three-dimensional geometric figures in a farm setting.

Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify two- and three-dimensional geometric figures, successfully calculate measurements of these figures, and be able to use the calculations to make comparisons and decisions in a real world setting.
- Describe planned time outline.
    - Focus Activity and Discussion (Two Options)
      Depending on your students prior understanding of the unit concept and setting choose one of the two activities listed below.
        - Video (15 minutes)
          Find an appropriate length video that allows for class discussion of the geometric shapes seen within the real world setting of the video. This may be a farm setting if your students are not familiar with this scenario.

Some options include...
http://www.teachertube.com/viewVideo.php?video_id=24955
http://www.watchknowlearn.org/Video.aspx?VideoID=36007&CategoryID=1010
http://www.youtube.com/watch?v=K9L9I86N-xM&list=PL8C80DD82B20625BE

- What Shape Am I? (15 minutes)
  Think – Complete focus worksheet individually.
  Pair – Group students together to discuss.
  Share – Extend into class discussion.

- Student Exploration
  - Square vs. Round? (40-45 minutes)
    In small groups have students work through the exploration packet.
- Classroom Discussion of Activity (15 minutes)
  - See questions below.
- Exit Slip Assessment (15 minutes)
- Assign homework (optional).

- Introduce the task.
  - You are spending the weekend with Farmer Fred down on the farm. He is asking you to help him make some decisions about the harvesting of his hay crop. All of these questions are geometrically related.
- Questions or prompts to pose.
  - What are the similarities between lateral area, surface area, and volume? The differences?
  - What types of geometric figures have lateral area, surface area, and volume?
  - How does a square bale get its' name?
  - Do size and shape matter? Why?
  - Which shape of bale do you prefer and why?

Student Exploration: Square vs. Round

Student/Teacher Actions:

- Students should use the formula sheet to calculate the required measurements given in the exploration. These measurements will be used to compare sizes of different three-dimensional geometric figures in the context of a real world setting.
- Teacher(s) will be guiding student groups as needed if questions/problems arise.
- It might be necessary to ask students to explain the difference between lateral area and surface area of a given three-dimensional geometric figure.

- Students will have 15 minutes to discuss results and ask questions in the classroom setting.

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties be providing models and/or pictures, clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to create their own mathematical question that Farmer Fred could need to know. This could be similar to the day's activity or not.

Assessment

- See the Rubric provided in Lesson 1.

Extension and Connections (for all students)

- Students are to connect to the theme/concept of geometry in a farm setting.
- Students are to connect geometric figures to specific objects and structures within a farm setting. (Also needed for tomorrow's activity.)
- Students will need to extend the concept of shape, size, and dimensions for activities in Lesson 4 and Lesson 5.

Strategies for Differentiation

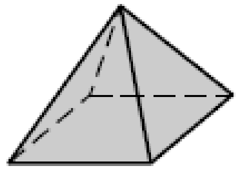
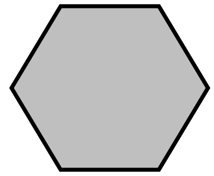
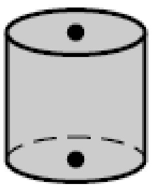
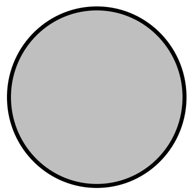
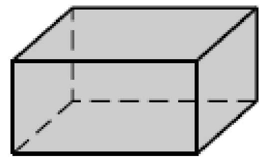
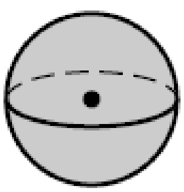
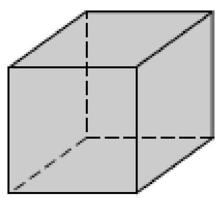
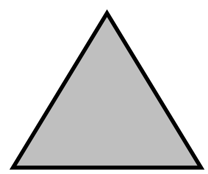
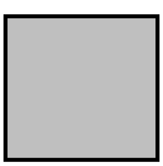
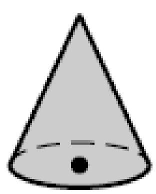
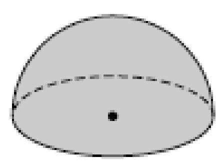
The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.

- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 1 – What Shape Am I? Name: ______________________________

Name each geometric figure. Classify it as 2-dimensional (2-D) or 3-dimensional (3-D).
List several places where you could see each figure in a real world setting.

 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:
 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:
 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:
 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:	 2-D or 3-D Name: Real World Setting:

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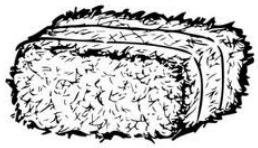
Student Exploration – Lesson 1 – Square vs. Round

NAME: _____

Scenario:
You are spending the weekend with Farmer Fred down on the farm. He is asking you to help him make some decisions about the harvesting of his hay crop. The questions below relate the geometry concept of two and three dimensional shapes to a real world setting. Please use the appropriate formula(s) from the given sheet to answer his questions.

1. Farmer Fred wants to know how much hay his square bales contain. Each bale measures $2ft \times 4ft \times 2ft$.



   a) What geometric shape best describes the bale?

   b) What measurement needs to be calculated? What formula needs to be used?

   c) Calculate the amount of hay in a single square bale.

   d) Calculate the amount of hay in all 211 bales of the farmer's harvest.

   e) Why do you think the farmer needs to know how much hay is in 211 bales?

2. Farmer Fred also harvests some of his hay crop in round bales and is curious how much each contains. Each round bale measures $5ft$ in diameter and $4ft$ deep.

   a) What geometric shape best describes the bale?

   b) What measurement needs to be calculated? What formula needs to be used?

   c) Calculate the amount of hay in a single round bale.

   d) Calculate the amount of hay in the 17 bales the farmer harvested today.

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3. Bale vs. Bale
    a) How many square bales would it take to equal just one round bale?

    b) How many round bales would the 211 square bales have made?

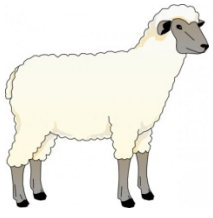
4. Around and around we go… Round bales are wrapped with twine to keep them from unrolling. Each bale is wrapped with twine numerous times. We will assume each bale is wrapped in twine 15 times.
    a) What measurement is being calculated? What formula needs to be used?

    b) How much twine is used to wrap one round bale?

5. Some round bales are completely covered in plastic to protect them from the weather.
    a) What measurement is being calculated? What formula needs to be used?

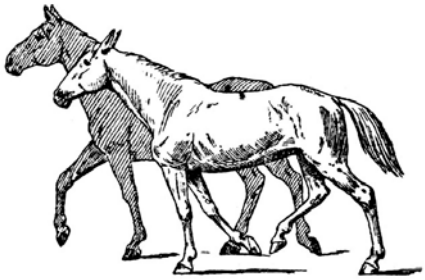
    b) How much plastic is needed to cover all 17 bales?

6. Dolly, Farmer Fred's pet sheep, eats 3.25 cubic feet of hay per day during the winter. What type of bale would be more realistic to use if only one bale is used per week with little left over? Please support your decision with the appropriate dimensions and calculations.



Exit Slip – Lesson 1 – Square vs. Round Name: _____

1. Farmer Fred's horses, Chester and Molly, also need to be fed hay during the winter. Chester eats 9 cubic feet of hay per day and Molly eats 7 cubic feet. If only one bale is used per week, what type of bale would be most realistic to feed both horses with little left over? Please support your decision with the appropriate dimensions of the bale type chosen.



2. Please write a paragraph summarizing your thoughts on today's activities. Be sure to discuss your challenges and any revelations you had during the lesson.

Homework – Lesson 1 – Square vs. Round Name: _____

Farmer Fred has decided to make his bales bigger…

1. The square bales are now going to be $3' \times 4' \times 3'$.
    a. What is the volume of one square bale?

    b. How much twine will be needed to tie the bale if two strands are used?

2. The round bales are now going to measure $5'$ in diameter and $5'$deep.
    a. What is the volume of one round bale?

    b. How much plastic is needed to completely cover one bale?

3. How many square bales are in a single round bale?

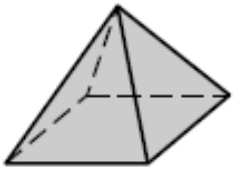
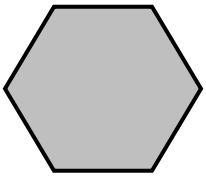
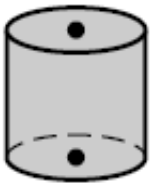
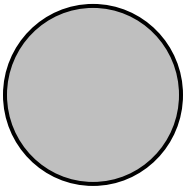
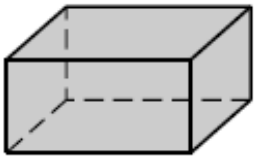
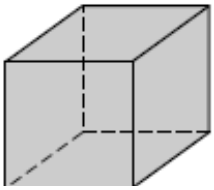
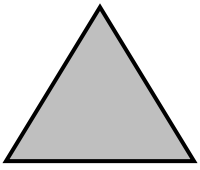
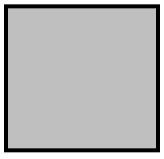
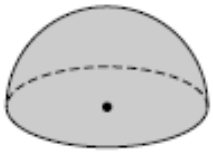
RUBRIC

(To be used on a daily basis.)

Category	*3*	*2*	*1*
Participation in Exploration	Participates in **ALL** parts of the exploration and class discussion.	Participates in **MOST** parts of the exploration and class discussion.	Little participation in the exploration activity and class discussion.
Exit Slip	Answers question(s) with process, calculations, or justification.	Answers question(s) with partial process, calculations, or justification.	Answers question(s) with no process, calculations, or justification.
Reflection ?(s)	Complete paragraph. Clearly answers and reflects on the day's activities.	Incomplete paragraph. Clearly answers and reflects on the day's activities.	Incomplete paragraph and incomplete thoughts/reflections on the day's
Homework (Optional)	Answers question(s) with process, calculations, or justification.	Answers question(s) with partial process, calculations, or justification.	Answers question(s) with no process, calculations, or justification.

Focus Activity – Lesson 1 – What Shape Am I? Name: _____Key_____

Name each geometric figure. Classify it as 2-dimensional (2-D) or 3-dimensional (3-D).
List several places where you could see each figure in a real world setting. Student answers will vary.

 **(2-D) or 3-D** Name: Rectangle Real World Setting: Window, door, table,tv/ computer screen	 2-D **or (3-D)** Name: Rectangular Pyramid Real World Setting: Roof of a house/barn,	 **(2-D) or 3-D** Name: Hexagon Real World Setting: Honeycomb,
 2-D **or (3-D)** Name: Cylinder Real World Setting: Food can,	 **(2-D) or 3-D** Name: Circle Real World Setting: Clock,	 2-D **or (3-D)** Name: Rectangular prism Real World Setting: shoe/food box,
 2-D **or (3-D)** Name: Sphere Real World Setting: Ball, globe, scoop of ice cream	 2-D **or (3-D)** Name: Cube Real World Setting: Box, Rubik's cube	 **(2-D) or 3-D** Name: Triangle Real World Setting: Yield sign,
 **(2-D) or 3-D** Name: Square Real World Setting: Window, floor tile	 2-D **or (3-D)** Name: Cone Real World Setting: Funnel, ice cream cone	 2-D **or (3-D)** Name: Hemisphere Real World Setting: Top of an ice cream cone

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Student Exploration – Lesson 1 – Square vs. Round Answer Key

1. Square Bales
   a) Square prism
   b) Volume. $V = l \times w \times h$
   c) $V = 2 \times 4 \times 2 = 16\ ft^3$
   d) $V = 16\ ft^3 \times 211 = 3{,}376\ ft^3$
   e) Farmer Jed will need to know if he has enough room to store his all his hay.
2. Round Bales
   a) Cylinder
   b) Volume. $V = \pi r^2 h$
   c) $V = \pi(2.5)^2(4) = 25\pi \approx 78.5\ ft^3$
   d) $V = 25\pi \times 17 = 425\pi \approx 1{,}335.2\ ft^3$
3. Bale vs. Bale
   a) $25\pi\ ft^3/16\ ft^3 \approx 4.9\ bales$
   b) $3{,}376\ ft^3/25\pi\ ft^3 \approx 43\ bales$
4. Around and around we go
   a) Circumference. $C = 2\pi r$
   b) $C = 2\pi r = 2\pi(2.5) = 5\pi \times 15 = 75\pi\ ft \approx 235.6\ ft$
5. Plastic wrap
   a) Surface Area. $SA = 2\pi r^2 + 2\pi rh$
   b) $SA = 2\pi(2.5)^2 + 2\pi(2.5)(4) = 32.5\pi \times 17 \approx 1{,}735.7\ ft^2$
6. $3.25 \times 7\ days = 22.75\ ft^3$
   Square bale
   Student answers will vary. Example: $V = 2 \times 2 \times 6 = 24\ ft^3$

Exit Slip – Lesson 1 – Square vs. Round Answer Key

1. Hay for horses
   $16 \times 7\ days = 112\ ft^3$
   Round bale
   Student answers will vary. Example: $V = \pi(3)^2(4) \approx 113.1\ ft^3$
2. Student reflections will vary.

Homework (optional) – Lesson 1 – Square vs. Round Answer Key

1. Square bale
   a) $V = 3 \times 4 \times 3 = 36\ ft^3$
   b) $P = 4 + 3 + 4 + 3 = 14 \times 2 = 28\ ft$
2. Round bale
   a) $V = \pi(2.5)^2(5) = 31.25\pi \approx 98.2\ ft^3$
   b) $SA = 2\pi(2.5)^2 + 2\pi(2.5)(5) = 37.5\pi \approx 117.8\ ft^2$

3. $98.2\ ft^3/36\ ft^3 \approx 2.7\ bales$

Lesson 2 – A Pen for Arnold

Strand
Geometry -- Three Dimensional (3-D) Figures

Mathematical Objective(s)
- Students will differentiate between the measurements of perimeter/circumference, area, lateral area, surface area, and volume.
- Students will relate the above measurements to the appropriate two- or three-dimensional figure(s) in a real-world setting and apply the use of appropriate units.
- Students will calculate the perimeter/circumference, area, lateral area, surface area, and volume of two- and three-dimensional figures using formulas in real-world applications.
- Students will compare measurements between different geometric figures (both different types and different sizes).
- Students will use manipulatives to construct a polygon that will maximize area.

Mathematics Performance Expectation(s)
- MPE.6 – The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- MPE.7 – The student will use similar geometric objects in two- or three-dimensions to
    - b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
    - d) solve real world problems about similar geometric objects.

Related SOL
- G.13 & G.14

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Draw and construct representations of two- and three- dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

Materials/Resources
- Classroom Set of Graphing Calculators

- Copies of Focus Activity (Measurements and Units)
- Copies of Student Exploration Packet (A Pen for Arnold)
- Copies of Assessment (Exit Slip – Lesson 2 – A Pen for Arnold)
- Copies of HW Assignment (optional).
- Set of 3-D models. (Cylinder, Triangular Prism, Rectangular Prism, Sphere, Hemisphere, Pyramid, Cone, Cube)
- Manipulatives (Pieces of skewers, straws, toothpicks, etc. – to represent lengths of $2'$, $3'$, $4'$, $5'$, and $10'$ for students who wish to build the structure instead of "graph" it; Geoboards with rubber bands could also be used)
- Copies of Geometry SOL Formula Sheet.
  www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf

Assumption of Prior Knowledge

- Students should be able to draw on prior knowledge of both two-dimensional and three-dimensional geometric figures.
- Students should be able to identify geometric figures by appearance even when combined. Students should be operating on the "analysis" level of the Van Hiele scale with respect to two- and three-dimensional geometric figures.
  - Students can recognize and name properties of the figures.
  - Properties cannot be tied to other properties that exist.
  - Students can reason informally about the figures.
- Students will begin to compare measurements of two non-similar three-dimensional geometric figures. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not know how to properly name prisms and will not be able to differentiate between lateral area, surface area, and volume.
- A priority for this lesson is for students to understand how to calculate perimeter/circumference, area, lateral area, and surface area and explain the differences in the measurements formed.
- Relevant contexts: Analysis and impact of sizes and shapes of three-dimensional geometric figures in a farm setting.

Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify two- and three-dimensional geometric figures, successfully calculate measurements of these figures, and be able to use the calculations to make comparisons and decisions in a real world setting.
- Describe planned time outline.

- Focus Activity and Discussion
  - Measurement and Units (10 minutes)
    - Students should circle all appropriate units for each measurement given.
    - Discuss what types of figures have LA, SA, V. This can be helpful with units.
- Student Exploration
  - "A Pen for Arnold" (45-50 minutes)
    - Students should explore with multiple figures to find the best pen shape and size for Arnold based on the specifications.
    - Students may opt to use manipulatives or not.
- Classroom Discussion of Activity (15 minutes)
  - See questions below.
- Exit Slip Assessment (15 minutes)
- Assign HW (optional).

- Introduce the task.
  - Farmer Fred wants to build a pen for his pig Arnold. The barn is a combination of two geometric shapes, the bottom is a rectangular prism and the top is a triangular prism as shown. The activity asks students in to calculate certain measurements in a real world setting.
  - This is extended to a bigger structure, a barn, in the optional homework assignment.
- Questions or prompts to pose.
  - What are the similarities between lateral area, surface area, and volume? The differences?
  - Why is the area of the base needed to calculate the volume of a prism?
  - Is the base needed to calculate the volume of a cylinder? Why or why not?
  - How do you maximize area? What shape allows this?
  - How do you minimize corners? Make better use of your fencing?

Student Exploration: A Pen for Arnold

Student/Teacher Actions:

- Students should use the formula sheet to calculate the required measurements given in the exploration. These measurements will be used to determine the amount of materials needed to build the barn and determine the usefulness (capacity) of the barn.

- Teacher(s) will be guiding student groups as needed if questions/problems arise, but will not answer the questions for the students.
- It might be necessary to ask students to explain the difference between lateral area and surface area of a given three-dimensional geometric figure.
- It might be necessary to ask student to explain the difference between the surface area and the volume of a given three-dimensional geometric figure.
- Students will have 15 minutes to discuss results and ask questions in the classroom setting.

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties be providing models and/or pictures, clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to create their own mathematical question that Farmer Fred could need to know. This could be similar to the day's activity or not.

Assessment

- See the Rubric provided in Lesson 1.

Extension and Connections (for all students)

- Students are to connect to the theme/concept of geometry in a farm setting.
- Students are to connect geometric figures to specific objects and structures within a farm setting. (Also needed for tomorrow's activity.)
- Students will need to extend the concept of shape, size, and dimensions for activities in Lesson 4 and Lesson 5.

Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.

- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 2 – Measurement and Units

Name: _____

Units are an important part of real world measurements. They can be useful in helping you calculate perimeter, areas, and volume.

Below is a chart with possible units for specific measurements.

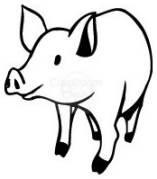
Please circle all reasonable units that could be used for each the measurement.

Measurement	Possible Units (circle all that apply)						
length	in^2	$miles^3$	$units^2$	cm	ft	mm^3	yd
surface area	ft^2	cm^2	yd^3	$miles$	mm^3	$units$	in^2
volume	mm	$units^3$	ft^2	in	$miles^2$	yd^2	cm^3
circumference	cm^3	yd	in^3	mm^2	$units$	ft^2	$miles$
lateral area	$miles^2$	in^3	mm	$units^3$	yd^3	cm	ft^2
perimeter	$units^2$	ft^3	cm	yd^2	in	$miles^2$	mm^3
area	yd^2	mm^2	$miles$	ft^3	cm^3	in	$units^2$

Student Exploration – Lesson 2 – A Pen for Arnold

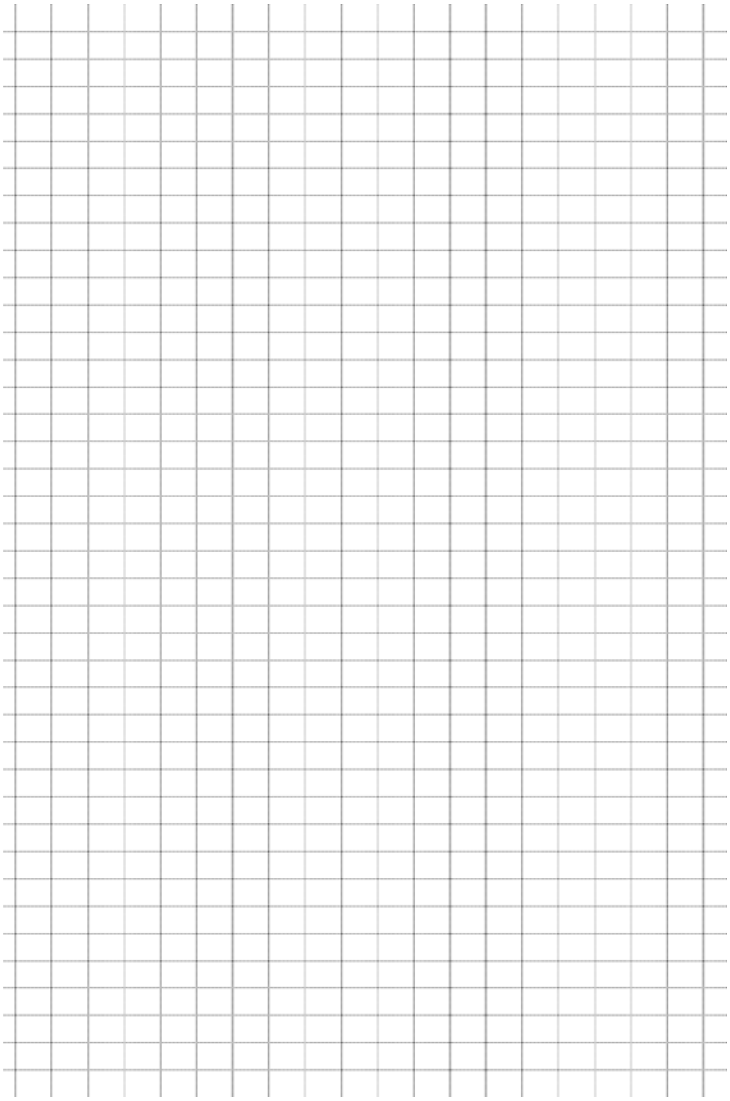
NAME: _____

Farmer Fred has 210 feet of fencing. He would like to build a pen for his pig, Arnold, near the barn. Can you help him decide how (shape and dimension) to build his pen?
The fence can be built using $2'$, $3'$, $4'$, $5'$, and $10'$ sections.



Using the given manipulatives of different lengths physically construct the boundary of the fence that satisfies the following criteria presented in each problem. Please record all of your "tries" in the space provided by giving a detailed sketch along with the necessary calculations to support your decision.

1. What shape and dimension(s) should Farmer Fred use to maximize the area of Arnold's pen?



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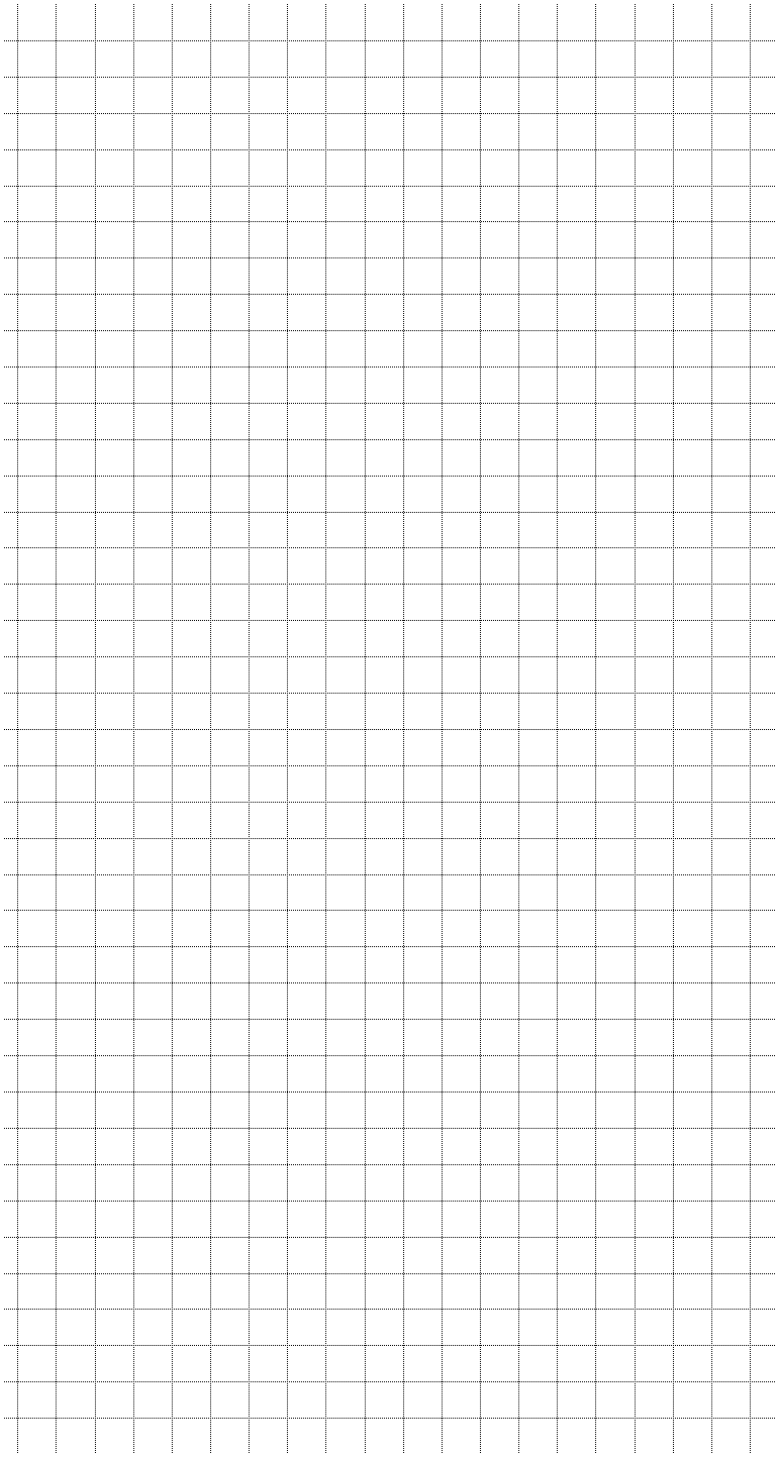
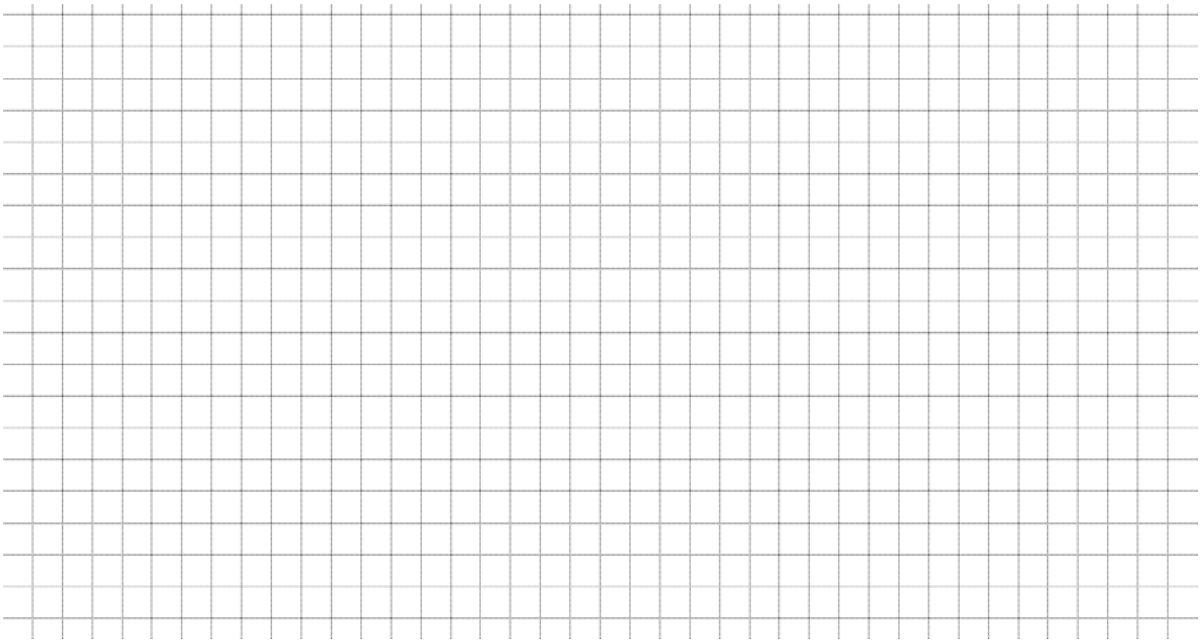
in collaboration with
Mr. Michael Bolling, Virginia Department of Education

2. What shape and dimension(s) should Farmer Fred use to minimize the number of corners in the fence?



Exit Slip – Lesson 2 – A Pen for Arnold Name: _____

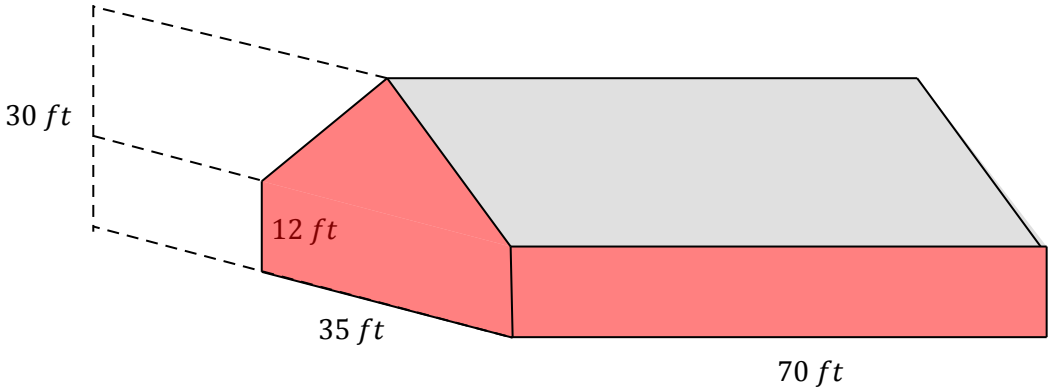
1. Using what you have already learned today is there a shape and dimension(s) that Farmer Fred could use to both maximize the area of Arnold's pen and minimize the number of corners in the fence?



2. Please write a paragraph summarizing your thoughts on today's activities. Be sure to discuss your challenges and any revelations you had during the lesson.

Homework – Lesson 2 – A Pen for Arnold Name: _____

Farmer Fred wants to build a barn for his pig Arnold. The barn is a combination of two geometric shapes, the bottom is a rectangular prism and the top is a triangular prism as shown.



1. Sketch each of the three-dimensional geometric figures separately. Be sure to label each with given measurements in feet.

2. How many square feet of siding will Farmer Fred need to purchase for the walls of his barn? Please specify what you are calculating and show the necessary steps to justify your answer.

3. Farmer Fred would like to paint the siding of his barn red. One gallon of paint will cover 85 square feet. To do a good job it will take two coats of paint. Each gallon of paint costs $22.50, how much would this cost Farmer Fred? Please show the necessary steps to justify your answer.

4. You calculated the volume of a square bale to be 16 cubic feet. How many square bales would fit in the loft (top) of the barn? Is the number realistic? Why or why not? Please justify your answers.

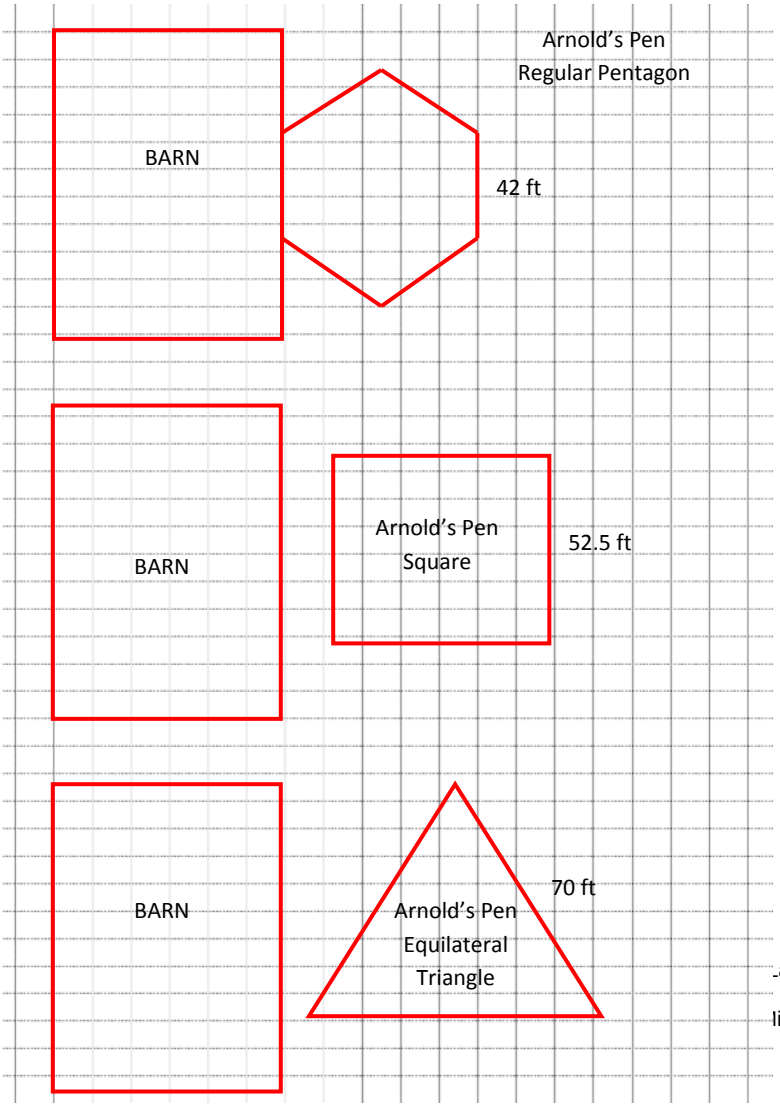
5. You calculated the volume of a round bale to be 25π cubic feet. How many round bales would fit in the bottom of the barn? Is this number realistic? Why or why not? Please justify your answers.

6. Which type of bale would "fit" better into the barn? Why?

Focus Activity – Lesson 2 – Measurement and Units Answer Key

Measurement	Possible Units (circle all that apply)						
Length	in^2	$miles^3$	$units^2$	**cm**	**ft**	mm^3	**yd**
surface area	**ft^2**	**cm^2**	yd^3	$miles$	mm^3	$units$	**in^2**
Volume	mm	**$units^3$**	ft^2	in	$miles^2$	yd^2	**cm^3**
circumference	cm^3	**yd**	in^3	mm^2	**$units$**	ft^2	**$miles$**
lateral area	**$miles^2$**	in^3	mm	$units^3$	yd^3	cm	**ft^2**
Perimeter	$units^2$	ft^3	**cm**	yd^2	**in**	$miles^2$	mm^3
Area	**yd^2**	**mm^2**	$miles$	ft^3	cm^3	in	**$units^2$**

Student Exploration/Exit Slip – Lesson 2 – A Pen for Arnold Answer Key – Student answers will vary!



If attached to the barn:

$A = \frac{1}{2}(42)(21tan60) \times 6 \approx 4{,}583 ft^2$

Not attached to the barn:

$A = \frac{1}{2}(35)(17.5tan60) \times 6 \approx 3{,}183\ ft^2$

If attached to the barn:

$A = (70)^2 = 4{,}900\ ft^2$

If not attached to the barn:

$A = (52.5)^2 = 2{,}756.25\ ft^2$

If attached to the barn:

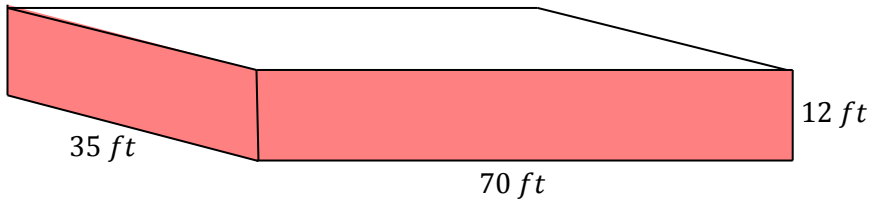
$A = \frac{1}{2}(105)(52.5tan60) \approx 4{,}774\ ft^2$

If not attached to the barn:

$A = \frac{1}{2}(70)(35tan60) \approx 2{,}122\ ft^2$

Homework – Lesson 2 – A Pen for Arnold Answer Key

1. Rectangular Prism and Triangular Prism



2. Need to calculate "Lateral Area" of the rectangular prism (or sum of rectangular sides) and the area of the two triangular ends.

$$LA = 2(12)(70) + 2(12)(35) = 2{,}520\ ft^2$$
$$A = \frac{1}{2}(18)(35) = 315\ \times 2 = 630\ ft^2$$

Farmer Fred would need to purchase $3{,}150\ ft^2$ of siding for the barn.

3. $3{,}150\ ft^2 \div 85 \approx 37\ gal \times \$22.50 = \$833.82 \times 2 = \$1667.65$

4. Square bales in loft

$$V = \left(\frac{1}{2} \cdot 35 \cdot 18\right)(70) = 22{,}050 ft^3 \div 16 \approx 1{,}378\ bales$$

Because of the wasted space between the square bales and the slant in the barn roof this number of square bales is not realistic.

5. Round bales in bottom.

$$V = (12 \cdot 35 \cdot 70) = 29{,}400 ft^3 \div 25\pi \approx 374\ bales$$

Because of the wasted space between the round bales and the walls as of the barn as well as between the round bales themselves this number of round bales is not realistic.

6. Student answers will vary.
The square bales would fit best in the bottom of the barn since they both represent rectangular prisms.

Lesson 3 – Which is Bigger?

Strand
Geometry -- Three Dimensional (3-D) Figures

Mathematical Objective(s)
- Students will differentiate between the measurements of area, lateral area, surface area, and volume.
- Students will relate the above measurements to the appropriate two- or three-dimensional figure(s) in a real-world setting.
- Students will calculate the perimeter/circumference, area, lateral area, surface area, and volume of two- and three-dimensional figures using formulas in real-world applications.
- Students will compare measurements between different geometric figures.

Mathematics Performance Expectation(s)
- MPE.6 – The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

- MPE.7 – The student will use similar geometric objects in two- or three-dimensions to
    - b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
    - d) solve real world problems about similar geometric objects.

Related SOL

- G.13 & G.14

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Draw and construct representations of two- and three- dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

Materials/Resources
- Classroom Set of Graphing Calculators
- Copies of Focus Activity (Which is Bigger?)
- Copies of Student Exploration Packet (Which is Bigger?)

- Cardstock (Two pieces per group cut into $8.5\ in \times 11\ in,\ 8\ in \times 14\ in, 6\ in \times 7\ in$)
- Roll of Tape (one per group)
- Copies of Assessment (Exit Slip – Lesson 3 – Which is Bigger?)
- Set of 3-D models (several different size cylinders)
- Copies of Geometry SOL Formula Sheet.
  www.doe.virginia.gov/testing/test_administration/ancilliary_materials/mathematics/2009/2009_sol_formula_sheet_geometry.pdf

Assumption of Prior Knowledge

- Students should be able to draw on prior knowledge of both two-dimensional and three-dimensional geometric figures.
- Students should be able to identify geometric figures by appearance even when combined. Students should be operating on the "analysis" level of the Van Hiele scale with respect to two- and three-dimensional geometric figures.
  - Students can recognize and name properties of the figures.
  - Properties cannot be tied to other properties that exist.
  - Students can reason informally about the figures.
- Students will compare measurements of two non-similar three-dimensional geometric figures. They should begin to discuss why these matter in a real world setting.
- It is likely that students will not be able to differentiate between lateral area, surface area, and volume.
- A priority for this lesson is for students to understand how to the dimension(s) of a three-dimensional geometric figure can affect the other measurements of the figure.
- Relevant contexts: Analysis and impact of sizes and shapes of three-dimensional geometric figures in a farm setting.

Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will construct a cylinder, successfully calculate measurements of these figures, and be able to use the calculations to make comparisons and decisions in a real world setting.
- Describe planned time outline.
  - Focus Activity and Discussion
    - Which is Bigger? (15 minutes)
      Think – Complete focus worksheet individually.
      Pair – Group students together to discuss.
      Share – Groups can share their answers. Teacher should not choose one over the others. (Might give away exploration goal!)

- Student Exploration
  - "Which is Bigger?" (45 minutes)
    Students work in small groups to complete the exploration worksheet.
- Classroom Discussion of Activity (15 minutes)
  - See questions below.
- Exit Slip Assessment (15 minutes)

- Introduce the task.
  - The main structure of a silo is a cylinder. Farmer Fred really needs a silo to store grain and corn in during the winter months. He is wondering about the overall dimensions of the silo. Should it be short and wide or tall and skinny? Please complete the exploration below to help Farmer Fred decide which dimension matters the most.
- Questions or prompts to pose.
  - What are the similarities between lateral area, surface area, and volume? The differences?
  - How can you use the dimensions of the paper to find the radius?
  - Does size really matter? Why or why not?
  - Which dimension, radius or height, affects the volume more?
  - Why does the lateral area remain the same in both "silos"?
  - Why does surface area not stay the same in both "silos?"

Student Exploration: Which is Bigger?

Student/Teacher Actions:

- Students should use the formula sheet to calculate the required measurements given in the exploration. These measurements will be used to determine the amount of materials needed to build the barn and determine the usefulness (capacity) of the barn.
- Teacher(s) will be guiding student groups as needed if questions/problems arise, but will not answer the questions for the students.
- It might be necessary to ask students to explain the difference between lateral area and surface area of a given three-dimensional geometric figure.
- It might be necessary to ask student to explain the difference between the surface area and the volume of a given three-dimensional geometric figure.
- Students will have 15 minutes to discuss results and ask questions in the classroom setting.

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties be providing models and/or pictures, clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to create a similar mathematical question about a geometric figure other than a cylinder that is present in a farm setting.

Assessment

- See the Rubric provided in Lesson 1.

Extension and Connections (for all students)

- Students are to connect to the theme/concept of geometry in a farm setting.
- Students are to connect geometric figures to specific objects and structures within a farm setting.
- Students will need to extend the concept of shape, size, and dimensions for activities in Lesson 4 and Lesson 5.

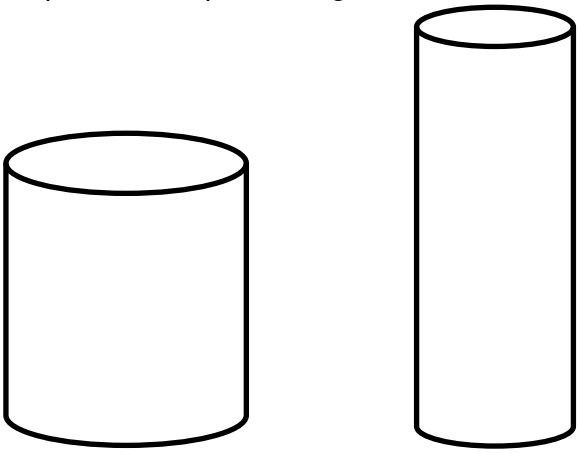
Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 3 – Which is Bigger? Name: _____

Consider the two following shapes without performing calculations.



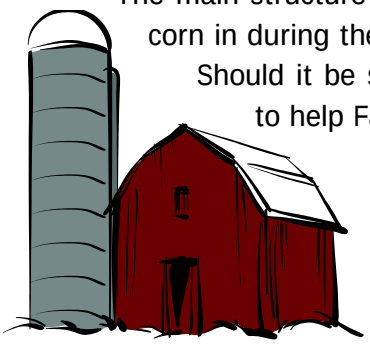
1. Which cylinder is bigger? Explain.

2. Which cylinder has the most surface area? Explain.

3. Which cylinder will hold the most (has the bigger volume)? Explain.

Student Exploration – Lesson 3 – Which is Bigger?

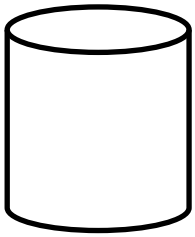
NAME: _____



The main structure of a silo is a cylinder. Farmer Fred really needs a silo to store grain and corn in during the winter months. He is wondering about the overall dimensions of the silo. Should it be short and wide or tall and skinny? Please complete the exploration below to help Farmer Fred decide which dimension matters the most.

In your groups take one sheet of your cardstock ($8.5\ in\ \times 11\ in,\ 8\ in\ \times 14\ in, 6\ in\ \times 7\ in$) and create a model that is a short, wide cylinder (silo) using the tape provided. The paper will form the shape of the cylinder, but it not a complete net. There will not be a top or bottom on the cylinder. *Justify your calculations with formulas and steps.*

1. Using the dimensions of the given cardstock calculate the height and radius of the cylinder.



2. Calculate the lateral area of the cylinder.

3. Calculate the surface area of the cylinder.

4. Calculate the volume.

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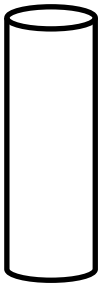
Now take the remaining sheet of cardstock and create a model that is a long, skinny cylinder (silo).



5. Using the dimensions of the given cardstock calculate the height and radius of the cylinder.

6. Calculate the lateral area of the cylinder.

7. Calculate the surface area of the cylinder.

8. Calculate the volume.

9. Please fill in the table. Be sure to include units in your calculations.

   Dimensions of the cardstock your group was given: _____

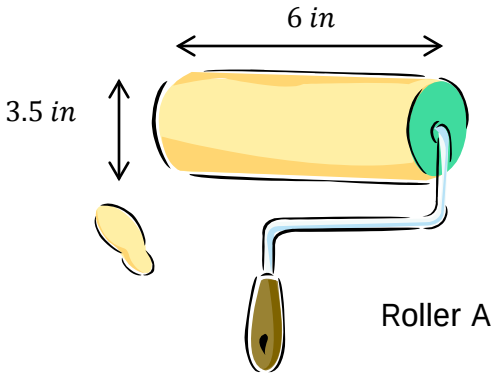
Short, Wide	Cylinder	Tall, Skinny
	Radius of Cylinder	
	Height of Cylinder	
	Lateral Area	
	Surface Area	
	Volume	

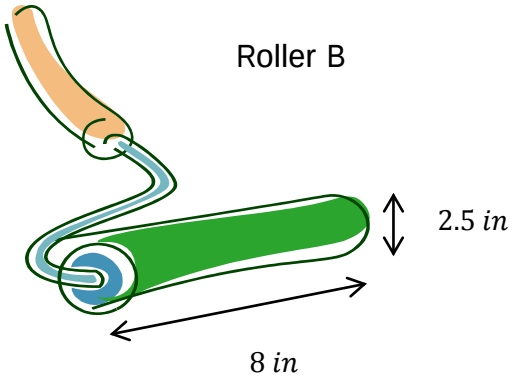
10. Write a paragraph comparing the measurements of the two cylinders.

11. When comparing the measurements of the two cylinders which measurement was most surprising to you? Why?

12. Which silo should Farmer Fred build… the short, wide cylinder or the long, skinny cylinder? Why?

Exit Slip – Lesson 3 – Which is Bigger? Name: _____





1. Since Farmer Fred wanted to paint the barn he was building for Arnold he needs to know which paint roller he should purchase. Which of the paint rollers above can spread move paint in a single stroke (revolution)? Justify your answer.

2. Please write a paragraph summarizing your thoughts on today's activities. Be sure to discuss your challenges and any revelations you had during the lesson.

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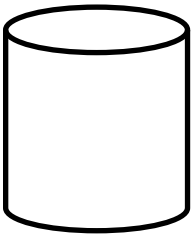
Focus Activity – Lesson 3 – Which is Bigger? Answer Key

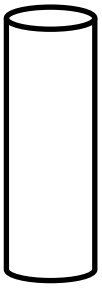
1. Student answers will vary.
2. Student answers will vary.
3. Student answers will vary.

Student Exploration – Lesson 3 – Which is Bigger? Answer Key

13. Calculate the lateral area of the cylinder.
    - $8.5 \times 11 - LA = 2\pi(5.5\pi)(8.5) = 93.5\pi^2 \approx 922.8\ in^2$
    - $8 \times 14 - LA = 2\pi(7\pi)(8) = 112\pi^2 \approx 1{,}105.4\ in^2$
    - $6 \times 7 - LA = 2\pi(3.5\pi)(6) = 42\pi^2 \approx 414.5\ in^2$
14. Calculate the surface area of the cylinder.
    - $8.5 \times 11 - SA = 2\pi(5.5\pi)^2 + 2\pi(5.5\pi)(8.5) \approx 2{,}798.7\ in^2$
    - $8 \times 14 - SA = 2\pi(7\pi)^2 + 2\pi(7\pi)(8) \approx 4{,}144\ in^2$
    - $6 \times 7 - SA = 2\pi(3.5\pi)^2 + 2\pi(3.5\pi)(6) \approx 1{,}174.2\ in^2$
15. Calculate the volume.
    - $8.5 \times 11 - V = \pi(5.5\pi)^2(8.5) = 257.125\pi^3 \approx 7{,}972.5\ in^3$
    - $8 \times 14 - V = \pi(7\pi)^2(8) = 392\pi^3 \approx 12{,}154.5\ in^3$
    - $6 \times 7 - V = \pi(3.5\pi)^2(6) = 73.5\pi^3 \approx 2{,}279\ in^3$



16. Calculate the lateral area of the cylinder.
    - $8.5 \times 11 - LA = 2\pi(4.25\pi)(11) = 93.5\pi^2 \approx 922.8\ in^2$
    - $8 \times 14 - LA = 2\pi(4\pi)(14) = 112\pi^2 \approx 1{,}105.4\ in^2$
    - $6 \times 7 - LA = 2\pi(3\pi)(7) = 42\pi^2 \approx 414.5\ in^2$
17. Calculate the surface area of the cylinder.
    - $8.5 \times 11 - SA = 2\pi(4.25\pi)^2 + 2\pi(4.25\pi)(11) \approx 2{,}042.9\ in^2$
    - $8 \times 14 - SA = 2\pi(4\pi)^2 + 2\pi(4\pi)(14) \approx 2{,}097.6\ in^2$
    - $6 \times 7 - SA = 2\pi(3\pi)^2 + 2\pi(3\pi)(7) \approx 972.6\ in^2$
18. Calculate the volume.
    - $8.5 \times 11 - V = \pi(4.25\pi)^2(11) \approx 6{,}160.6\ in^3$
    - $8 \times 14 - V = \pi(4\pi)^2(14) = 224\pi^3 \approx 6{,}945.4\ in^3$
    - $6 \times 7 - V = \pi(3\pi)^2(7) = 63\pi^3 \approx 1{,}953.4\ in^3$



19. See measurements above.
20. Student answers will vary.
21. Student answers will vary.
22. Student answers will vary.

Exit Slip – Lesson 3 – Which is Bigger? Answer Key

1. $LA = 2\pi(1.75)(6) = 21\pi \approx 66\ in^2$ -- Will spread more paint in a single stroke
   $LA = 2\pi(1.25)(8) = 20\pi \approx 63\ in^2$
2. Student answers will vary.

Lesson 4 – Which of these is like the other?

Strand
Geometry -- Three Dimensional (3-D) Figures

Mathematical Objective(s)
- Students will identify similar three-dimensional geometric figures and show the scale factor (proportion) that exists between dimensions of these figures.
- Students will solve proportions.

Mathematics Performance Expectation(s)
- extension of MPE.34 – The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as proofs.

Related SOL
- G.14

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Draw and construct representations of two- and three- dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

Materials/Resources
- Classroom Set of Graphing Calculators
- Copies of Focus Activity (Proportions and Scale)
- Copies of Student Exploration Packets (Which of these is like the other?, Constructing Similar 3-D Figures)
- Sheets of cardstock
- Copies of nets. http://edgalaxy.com/journal/2012/8/5/great-collection-of-3d-shape-nets-to-download-and-print.html
- Copies of Assessment (Exit Slip – Lesson 4 – Which of these is like the other?)
- Set of 3-D models. (Cylinder, Triangular Prism, Rectangular Prism, Sphere, Hemisphere, Pyramid, Cone, Cube)

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Assumption of Prior Knowledge

- Students should be able to identify similar figures by appearance and solve proportions. Students should be operating on the "analysis" level of the Van Hiele scale with respect to similar two- and three-dimensional geometric figures.
    - Students can recognize and name properties of the figures.
    - Properties cannot be tied to other properties that exist.
    - Students can reason informally about the figures.
- Students should begin to realize that special relationships exist between similar figures. They should begin to discuss what these properties are and how to verify that figures are indeed similar.
- It is likely that students will not know how to solve proportions with a variable in more than one place.
- A priority for this lesson is for students to understand similar figures and the relationships that exist between similar figures.
- Relevant contexts: Analysis and impact of sizes and shapes of three-dimensional geometric figures in a farm setting.

Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify two- and three-dimensional geometric figures that are similar and determine the property (ratio) that make them similar.
- Describe planned time outline.
    - Focus Activity and Discussion
        - Proportions & Scale (10 minutes)
        - Remind students about the distributive property.
    - Student Exploration #1
        - "Which of these is like the other?" (25-30 minutes)
          Think – Complete focus worksheet individually.
          Pair – Group students together to discuss.
          Share – Extend into class discussion.
    - Student Exploration #2
        - "Constructing Similar 3-D Figures" (30 minutes)
          Students will be given the net of at least one 3-D geometric figure.
          http://edgalaxy.com/journal/2012/8/5/great-collection-of-3d-shape-nets-to-download-and-print.html
          From this net students will construct a similar geometric solid.
          Students should "build" both models to verify their similarity.

    - Classroom Discussion (10 minutes)
        - See questions below. These could have been brought up in the Think-Pair-Share activity above.
    - Exit Slip Assessment (10 minutes)
- Introduce the task.
    - There are many similar shapes that exist in real world settings including a farm. Use your knowledge to decide if the pairs of figures below are indeed similar. If so, state the scale factor that exists. If not, explain why the figures are not similar. We will use our knowledge of similar figures to learn about the relationship between three-dimensional geometric figure and their measurements tomorrow.
- Questions or prompts to pose.
    - Are similar figures congruent?
    - Are congruent figures similar?
    - What properties are present in similar three-dimensional figures?
    - Why are cylinders and prisms not similar figures?
    - Why are cones and pyramids not similar figures?
    - Why are cones and cylinders not similar figures?
    - Why are pyramids and prisms not similar figures?
    - Were the silos constructed during Lesson 3 similar? Why or why not?

Student Exploration #1: Which of these is like the other?

Student/Teacher Actions:

- Students should decide if the figures given are similar and show justification if they are similar or be able to explain why they are not similar.
- Teacher(s) will be guiding student groups as needed if questions/problems arise, but will not answer the questions for the students.
- It might be necessary to ask students to explain the difference between congruent and similar figures.
- It might be necessary to ask students to explain the differences between cylinders and cones, cylinders and prisms, and prisms and pyramids.
- Students will have 10 minutes to work individually and 10 minutes to work in small groups, then 5-10 minutes to discuss results and ask questions in the classroom setting.

Student Exploration #2: Constructing Similar 3-D Figures

Student/Teacher Actions:

- Students will be given the net of at least one 3-D geometric figure.

- From this net students will construct a similar geometric solid.
- Students will have to carefully measure the “necessary” parts (radius, height, etc.) in order to scale the similar geometric solid.
- It might be necessary to ask students what dimensions really “matter” to the given shape.
- Students should “build” both models to verify their similarity.
- Students will have 10 minutes to discuss results and ask questions in the classroom setting.

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties be providing models and/or pictures, clarifying directions, and prompt students to the next step with leading questions.
- The teacher will extend the material for students that are ready to move forward by asking them to create a set of three similar geometric figures that could be found on a farm setting and a set of three non-similar geometric figures.

Assessment

- See the Rubric provided in Lesson 1.

Extension and Connections (for all students)

- Students are to connect to the theme/concept of geometry in a farm setting.
- Students will need to extend the concept of shape, size, and dimensions for activities in Lesson 5.

Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student’s primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.

- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 4 – Proportions and Scale

Name: _____

Solve the proportions for the given variable. Please show your steps!

1. $\frac{x}{10} = \frac{15}{25}$

2. $\frac{4}{6} = \frac{18}{m}$

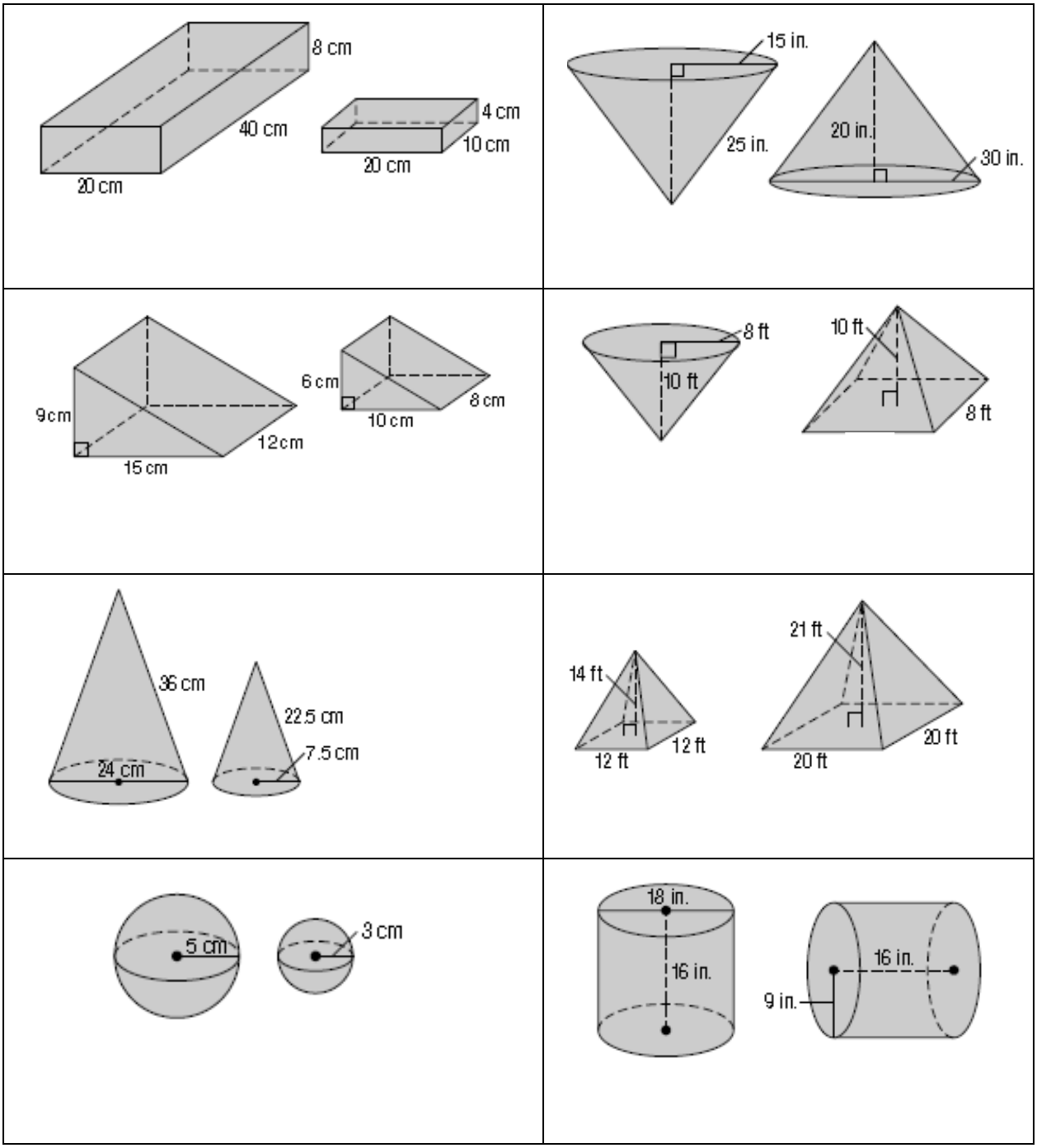
3. $\frac{y+3}{6} = \frac{5}{3}$

4. $\frac{4}{w} = \frac{8}{w+4}$

Student Exploration #1– Lesson 4 – Which of These is Like the Other?

NAME: _____

There are many similar shapes that exist in real world settings including a farm.
Use your knowledge to decide if the pairs of figures below are indeed similar.
If so, state the scale factor that exists. If not, explain why the figures are not similar.



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Student Exploration #2– Lesson 4 – Constructing Similar 3-D Figures

NAME: _____

Using the net you were given...

1. Carefully consider the 3-D figure it forms to determine its' important dimensions.
2. Carefully measure the net.
3. Using your measurements determine appropriate measurements for a figure similar to the one in the net provided.
4. Create your own net using these "new" dimensions on a sheet of cardstock.
5. Construct both figures from their nets.
6. Answer the following questions.
    a. Do they appear similar?
    b. If so, why?
    c. If not, what are some possible reasons they are not similar?

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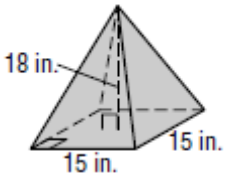
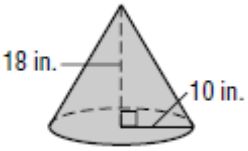
Exit Slip – Lesson 4 – Which of These is Like the Other?

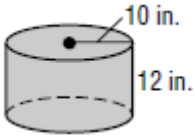
Name: _____

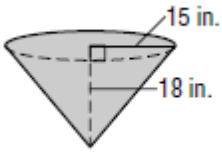
1. What properties do similar figures have?

2. Which of the following shapes are similar? Circle all that apply. Justify your answer(s).









3. Please write a paragraph summarizing your thoughts on today's activities. Be sure to discuss your challenges and any revelations you had during the lesson.

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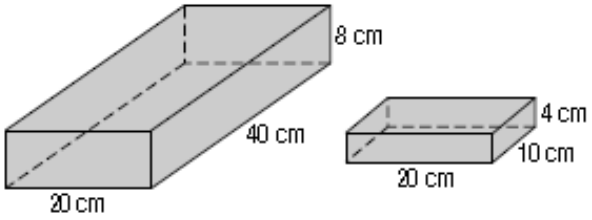
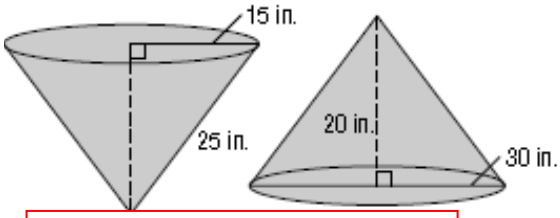
Focus Activity – Lesson 4 – Proportions and Scale Answer Key

1. $x = 6$
2. $m = 27$
3. $y = 7$
4. $w = 4$

Student Exploration – Lesson 4 – Which of These is Like the Other? Answer Key

<table>
<tr>
<td> 

$Similar; scale\ factor\ 2:1$</td>
<td> 

$Not\ Similar; \frac{15}{25} \neq \frac{15}{20}$</td>
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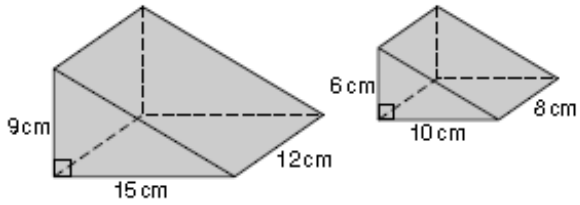
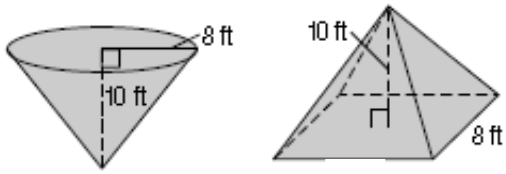
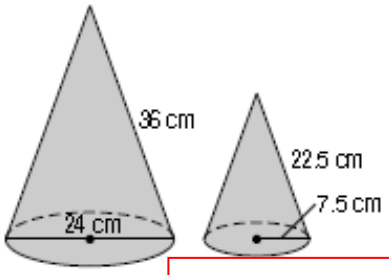
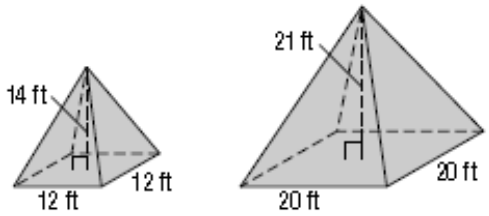
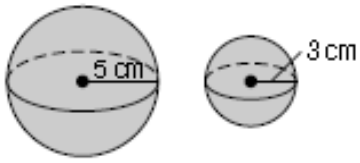
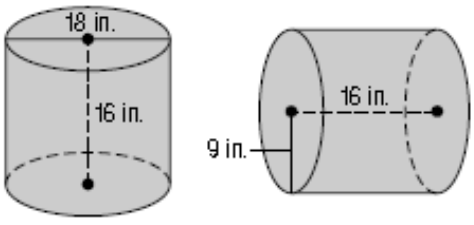
<td> 

$Similar; scale\ factor\ 3:2$</td>
<td> 

$Not\ Similar;$ Not the same shape</td>
</tr>
<tr>
<td> 

$Similar; scale\ factor\ 8:5$</td>
<td> 

$Not\ Similar; \frac{14}{12} \neq \frac{21}{20}$</td>
</tr>
<tr>
<td> 

$Similar; scale\ factor\ 5:3$</td>
<td> 

$Similar(Congruent); scale\ factor\ 1:1$</td>
</tr>
</table>

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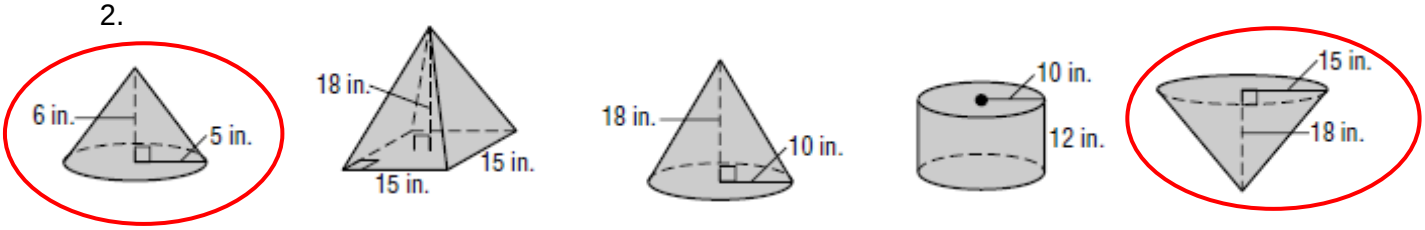
Exit Slip – Lesson 4 – Which of These is Like the Other? Answer Key

1. Student answers may vary.
   Similar figures have the exact same shape but different (proportional) sizes.
   Similar figures corresponding measurements (side lengths, radius, height) are proportional.
2.



Read individual student responses.

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Lesson 5 – Similarity Ratios

Strand
Geometry -- Three Dimensional (3-D) Figures

Mathematical Objective(s)
- Students will compare measurements between different geometric figures.
- Students will determine and use the similarity ratio (scale) of a dimension of two similar three-dimensional geometric figures to also determine the similarity ratio of the surface area and volume of the figures and vice versa.
- Students will calculate the surface and volume of the similar figure using only the similarity ratio.

Mathematics Performance Expectation(s)
- MPE.6 – The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.
- MPE.7 – The student will use similar geometric objects in two- or three-dimensions to
  a) compare ratios between side lengths, perimeters, areas, and volumes;
  b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
  c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
  d) solve real-world problems about similar geometric objects.
- MPE.34 – The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as proofs.

Related SOL

- G.13, G.14, and extension of G.7

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects.
- Draw and construct representations of two- and three- dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections.

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Materials/Resources

- Classroom Set of Graphing Calculators
- Copies of Focus Activity (Similarity Ratios)
- Copies of Student Exploration Packet (Arnold & Chester)
- Copies of Assessment (Exit Slip – Lesson 5 – Similarity Ratios)
- Set of 3-D models. (Cube, sphere, cylinder)

Assumption of Prior Knowledge

- Students should be able to identify similar figures by appearance and solve proportions. Students should be operating on the "analysis" level on Van Hiele scale with respect to similar figures.
    - Students can recognize and name properties of the figures.
    - Properties cannot be tied to other properties that exist.
    - Students can reason informally about the figures.
- Students should begin to realize that special relationships exist between similar figures. They should begin to discuss what these properties are and how to verify that figures are indeed similar.
- It is likely that students will not realize that the similarity ratio between lengths (of dimensions) can be used to discuss the ratio between the surface areas and volumes of two similar three-dimensional geometric figures.
- A priority for this lesson is for students to understand how to use the similarity ratio (scale) of a dimension of two similar three-dimensional geometric figures to also determine the similarity ratio of the surface area and volume of the figures and vice versa.
- Relevant contexts: Analysis and impact of sizes and shapes of three-dimensional geometric figures in a farm setting.

Introduction: Setting Up the Mathematical Task

- Clearly introduce the goal of the lesson.
  In this lesson, the student will be able to identify two- and three-dimensional geometric figures that are similar and show the properties that make them similar.
- Describe planned time outline.
    - Focus Activity and Discussion
        - Similarity Ratios (10 minutes)
          Should every student's answer be the same?
          Discuss the concept of length versus depth.
    - Student Exploration
        - "Arnold & Chester?" (45-50 minutes)

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Students should work in pairs to complete the activity.
    - Classroom Discussion of Activity (15 minutes)
        - See questions below.
    - Exit Slip Assessment (15 minutes)
- Introduce the task.
    - Using the given items on the farm calculate the similarity ratio of the dimension given. Then calculate the specified measurement and that comparison ratio between the two items as well.
- Questions or prompts to pose.
    - What properties exist in similar figures?
    - If one dimension changes by a scale factor, do they all change by that same factor?
    - What relationship exists between the similarity ratios of length, area, and volume?
    - When solving literal equations how to you solve for a variable raised to a power?
    - Why is it useful to leave these calculations in terms of π?

Student Exploration: Arnold & Chester

Student/Teacher Actions:

- Students should calculate the requested information about each of the similar figures and then use those measurements to form similarity ratios.
- Teacher(s) will be guiding student groups as needed if questions/problems arise, but will not answer the questions for the students.
- It might be necessary to focus students' attention to the relationship between the similarity ratios of length, area, and volume.
- It might be necessary to remind students that similar figures are proportional – if one dimension is doubled all of the other dimensions are also doubled.
- Students will have 15 minutes to discuss results and ask questions in the classroom setting.

Monitoring Student Responses

- Students are expected to discuss the exploration activities together in their groups and then as a class at the end of the activity to provide feedback and reinforce the lesson.
- The teacher will assist students who have difficulties be providing models and/or pictures, clarifying directions, and prompt students to the next step with leading questions.

- The teacher will extend the material for students that are ready to move forward by asking them to find the surface area and volume of a third similar figure (bigger or smaller) of their making from the exploration activity. Students could also create an area and volume of a third figure and see how the dimensions of the shape changed.

Assessment

- See the Rubric provided in Lesson 1.

Extension and Connections (for all students)

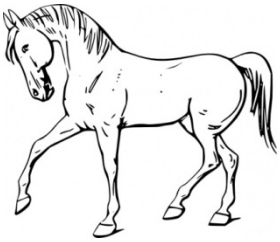
- Students are to connect to the theme/concept of geometry in a farm setting.
- Students will extend the concept from the day's activities to more easily calculate measurements of similar geometric figures (i.e. use the similarity ratio).

Strategies for Differentiation

The graphic organizers/worksheets/handouts were designed with the needs of a diverse classroom of students in mind. There is a visual representation of each situation. Tables were created to assist students as well. Use of the graphing calculator is also encouraged.

- For ELL learners, teachers should work with the ELL teacher to provide bridges between mathematics vocabulary and the student's primary language. ELL students could keep a vocabulary journal to assist them.
- Learning disabled students may benefit if the teacher provides multiple choice answers to the student explorations.
- Visual learners will benefit from the pictures provided on the activities.
- Auditory learners will benefit from the classroom and group discussions.
- Kinesthetic learners will benefit from movement from individual work to group work and the ability to work with models.
- High ability students may start to begin to compare the similarities or differences and offer opinions to lead into tomorrow's lessons. These students can also serve as peer leaders with groups that are struggling to complete the task(s).

Focus Activity – Lesson 5 – Similarity Ratios NAME: _____



Farmer Fred has a horse named Chester. Chester has a stall measuring $10\ ft$ long, $12\ ft$ deep, and $4\ ft$ tall. The farmer wants to build Arnold, his pet pig, a similar but obviously smaller stall in the barn at a scale of $3:1$ of Chester's existing stall. What would the dimensions of Arnold's stall be? Please justify your answer.

The isometric dot paper below is provided in case you would like to model the situation.

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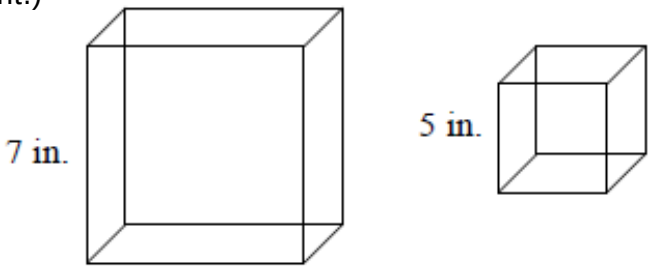
Student Exploration – Lesson 5 – Arnold & Chester

NAME: _____

Farmer Fred has two pets, Arnold the Pig and Chester the Horse. They have similar items. Can you help him compare his pets' toys, salt blocks, and feed cans? (Be sure to make a comparison ratio of each measurement.)

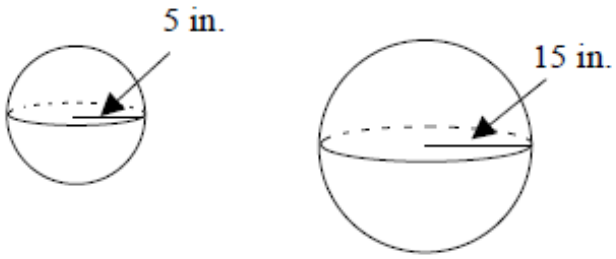
1.



Please include units with your calculations.

Chester's Salt Block	Cube	Arnold's Salt Block	Comparison Ratio
$7\ in$	Length	$5\ in$	
	Surface Area		
	Volume		

2.



Please include units with your calculations and leave in terms of π.

Arnold's Toy (Ball)	Sphere	Chester's Toy (Ball)	Comparison Ratio
$5\ in$	Radius	$15\ in$	
	Surface Area		
	Volume		

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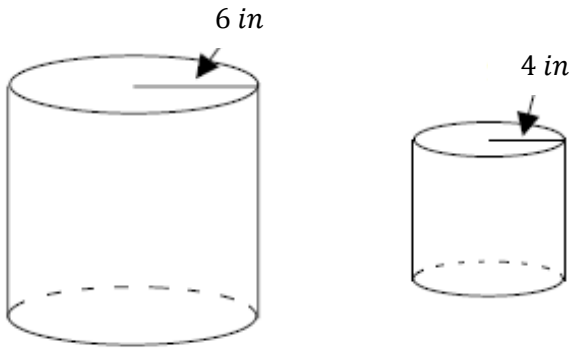
3. Is there anything you notice about the relationship between the ratio of radius/length and the surface area in the previous questions?

4. Is there anything you notice about the relationship between the ratio of radius/length and the volume in the previous questions?

5. Using the relationships you found in questions 3 and 4 fill in the following chart. (Remember that as long as the figures are similar then all dimensions must change by the same factor – you do not need to know the height of the cylinders!)



Please include units with your calculations and leave in terms of π.

Comparison Ratio	Cylinder	Chester's Feed Can	Arnold's Feed Can
	Length	$6\ in$	$4\ in$
	Surface Area	$216\pi\ in^2$	
	Volume		$128\pi\ in^3$

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Exit Slip – Lesson 5 – Similarity Ratios NAME: _____

1. Complete the table by finding the missing ratios.

Similarity (Length) Ratio	Area Ratio	Volume Ratio
1:3		
2:5		
	16:49	
		27:125

2. Please write a paragraph summarizing your thoughts on today's activities. Be sure to discuss your challenges and any revelations you had during the lesson.

3. Please write a paragraph discussing your opinions of this mathematical unit. What have you liked and disliked? What concepts have you mastered? What concepts are still unclear?

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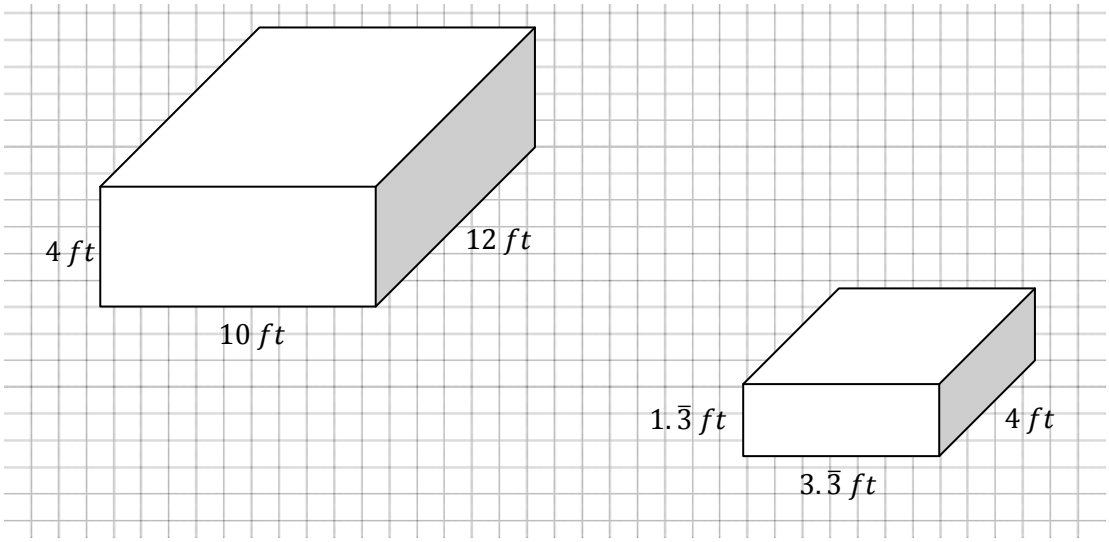
Focus Activity – Lesson 5 – Similarity Ratios Answer Key



Student Exploration – Lesson 5 – Arnold & Chester Answer Key

1.

Chester's Salt Block	Cube	Arnold's Salt Block	Comparison Ratio
$7\ in$	Length	$5\ in$	$\frac{7}{5}$
$SA = 2(7)(7) \times 3 = 294\ in^2$	Surface Area	$SA = 2(5)(5) \times 3 = 150\ in^2$	$\frac{294}{150} = \frac{49}{25}$
$V = (7)(7)(7) = 343\ in^3$	Volume	$V = (5)(5)(5) = 125\ in^3$	$\frac{343}{125}$

2.

Arnold's Toy (Ball)	Sphere	Chester's Toy (Ball)	Comparison Ratio
$5\ in$	Radius	$15\ in$	$\frac{5}{15} = \frac{1}{3}$
$SA = 4\pi(5)^2 = 100\pi$	Surface Area	$SA = 4\pi(15)^2 = 900\pi$	$\frac{1}{9}$
$V = \frac{4}{3}\pi(5)^3 = \frac{500}{3}\pi\ in^3$	Volume	$V = \frac{4}{3}\pi(15)^3 = 4500\pi\ in^3$	$\frac{1}{27}$

3. The ratio of the surface area should be the square of the radius/length.

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4. The ratio of the volume should be the cube of the radius/length.

5.

Comparison Ratio	Cylinder	Chester's Feed Can	Arnold's Feed Can
$\frac{6}{4}=\frac{3}{2}$	Length	$6\ in$	$4\ in$
$\frac{3^2}{2^2}=\frac{9}{4}$	Surface Area	$216\pi\ in^2$	96π
$\frac{3^3}{2^3}=\frac{27}{8}$	Volume	432π	$128\pi\ in^3$

Exit Slip – Lesson 5 – Similarity Ratios Answer Key

1.

Similarity (Length) Ratio	Area Ratio	Volume Ratio
1:3	$1:9$	$1:27$
2:5	$4:25$	$8:125$
$4:7$	16:49	$64:343$
$3:5$	$9:25$	27:125

2. Read individual student responses.
3. Read individual student responses.

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